

Quantum thermodynamics

— a primer for the curious quantum mechanic

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Outline for lectures at YQIS 2018, Vienna

(presented by Ralph Silva, ETH Zurich)



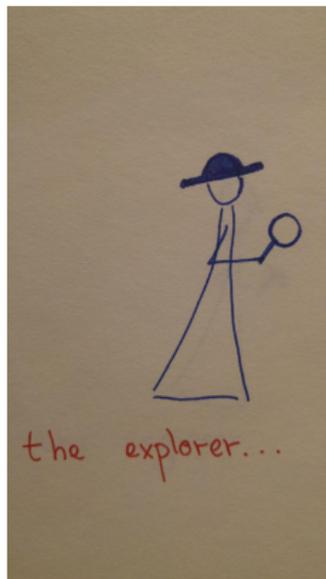
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Why quantum thermodynamics?



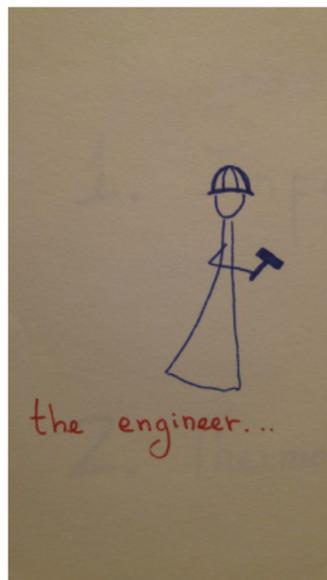
- ▶ Why is thermodynamics so effective?
- ▶ Emergent theory?
- ▶ Axiomatic formulation?

Why quantum thermodynamics?



- ▶ Do quantum systems obey the laws of thermodynamics?
- ▶ Correction terms: small or quantum?
- ▶ Can we explore new effects?

Why quantum thermodynamics?



- ▶ Heat dissipation in (quantum) computers
- ▶ Microscopic heat engines
- ▶ “Thermodynamics” of relevant parameters at the nano scale?

This lecture

Information and thermodynamics

- ▶ Work cost of classical information processing
- ▶ Quantum work extraction and erasure

Axiomatic quantum thermodynamics

- ▶ Resource theory of thermal operations
- ▶ Insights and results
- ▶ Directions

Maxwell's demon



(P, V)



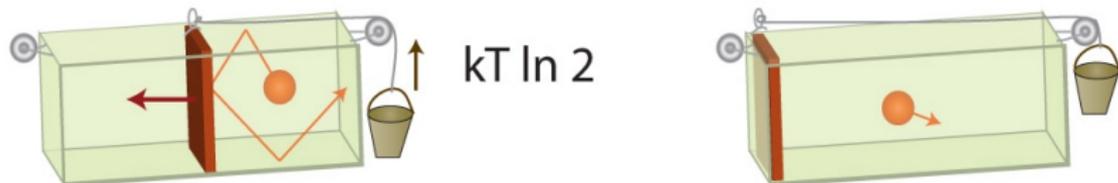
$\{(\vec{x}_i, \vec{p}_i)\}$

P_{demon}

Thermodynamics of information processing

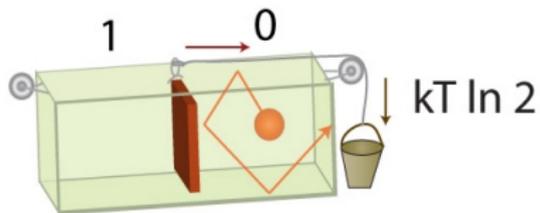
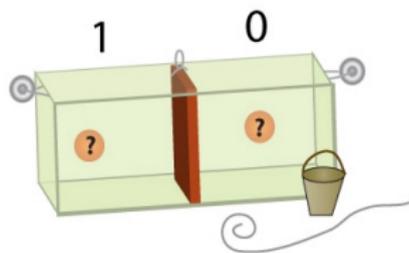
- ▶ How much work must we supply to compute a function?
- ▶ Must (quantum) computers always dissipate heat?

Szilard boxes



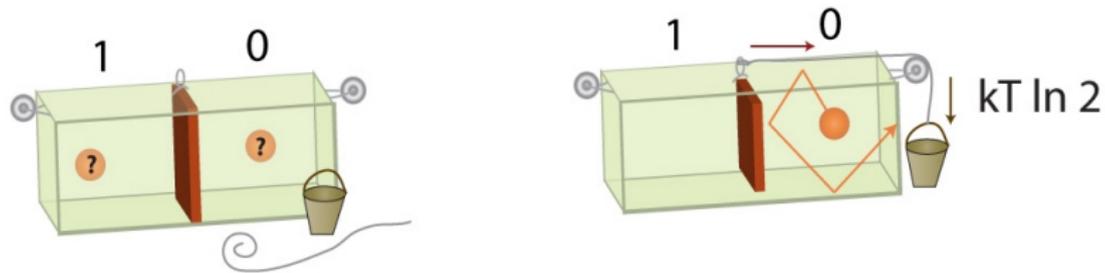
1 bit + heat bath (T) \Rightarrow work $kT \ln 2$

Szilard boxes



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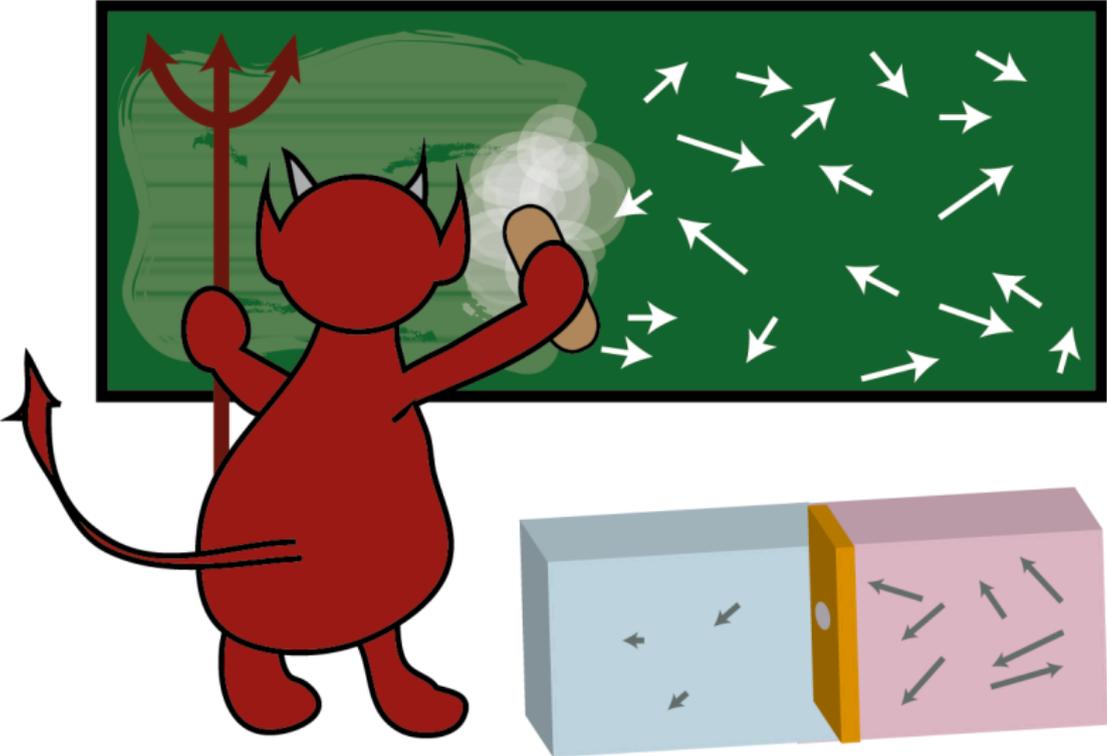
$$1 \text{ bit} + \text{heat bath } (T) \Leftrightarrow \text{work } kT \ln 2$$

Landauer's principle [1973]

Information + heat \Leftrightarrow work

► rate: $kT \ln 2$ per bit

Maxwell's demon



Cost of computations [Bennett 1992]

Must computers dissipate heat?

- ▶ Irreversible computation: reversible + erasure
- ▶ $\mathcal{E}(\rho_S) = \text{Tr}_{A'}(U \rho_S \otimes \sigma_A U)$

Cost of computations [Bennett 1992]

Must computers dissipate heat?

- ▶ Irreversible computation: reversible + erasure
- ▶ $\mathcal{E}(\rho_S) = \text{Tr}_{A'}(U \rho_S \otimes \sigma_A U)$
- ▶ Reversible computations: free in principle
- ▶ Work cost: cost of erasure

Work cost of erasure

$kT \ln 2$ per bit

Erasure

- ▶ Formatting a hard drive:

0?10101??1 \rightarrow 000000000

- ▶ Resetting a quantum system: $\rho_S \rightarrow |0\rangle_S$

Work cost of erasure

$$kT \ln 2 \text{ per bit}$$

Erasure

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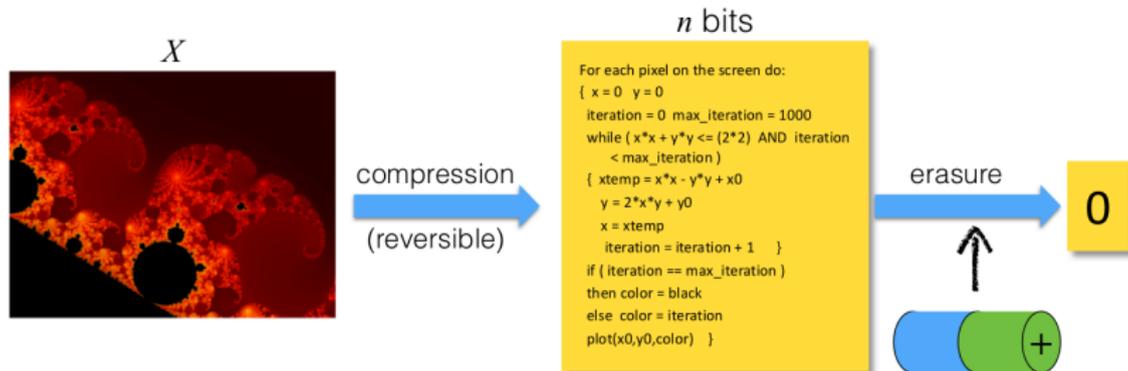
$$0?10101??1 \rightarrow 000000000$$

- ▶ Resetting a quantum system: $\rho_S \rightarrow |0\rangle_S$

In numbers

- ▶ $k = 1.38 \cdot 10^{-23} \text{ J/K}$
- ▶ Erasure of 16TB hard drive at room temperature: $0.4 \mu\text{J}$
- ▶ Lifting a tomato by 1m on Earth: 1J .

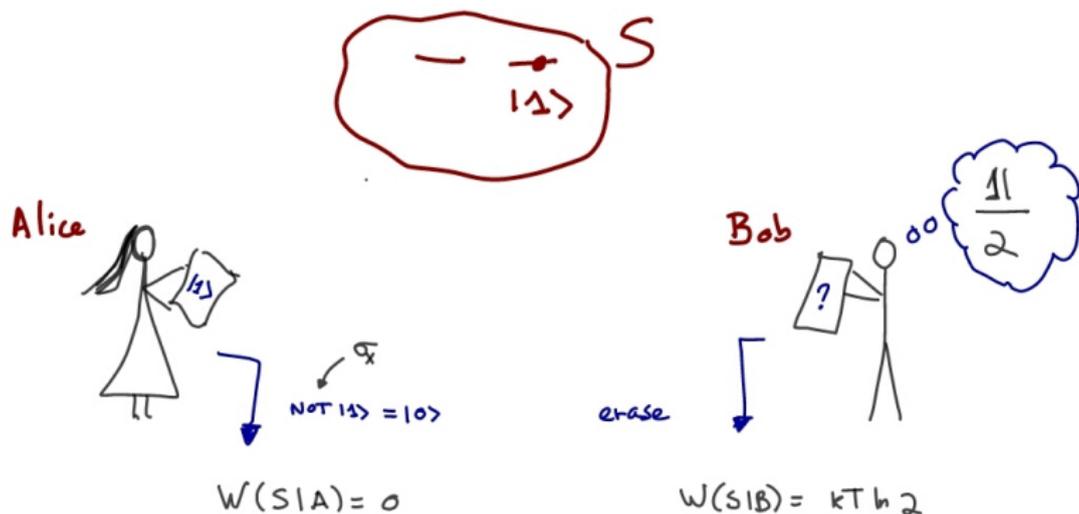
Information compression



Compression length: $n = H(\rho)$ bits

$$W(S) = H(S) kT \ln 2$$

(Subjective) side information



$$W(S|M) = H(S|M) kT \ln 2$$

What about quantum information?

- ▶ Szilard box for quantum systems?
 - ▶ How do we even measure work?

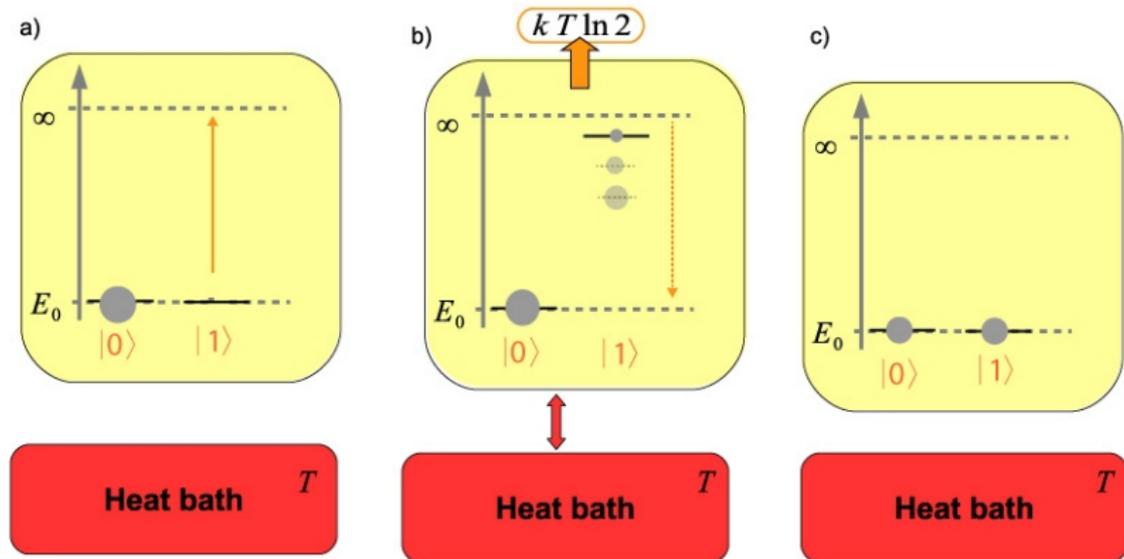
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- ▶ How to use quantum memories?
 - ▶ Reading \implies disturbing contents

What about quantum information?

- ▶ Szilard box for quantum systems?
 - ▶ How do we even measure work?
- ▶ How to use quantum memories?
 - ▶ Reading \implies disturbing contents
- ▶ Entropy $H(S|M)$ can be negative!
 - ▶ but does that mean anything?

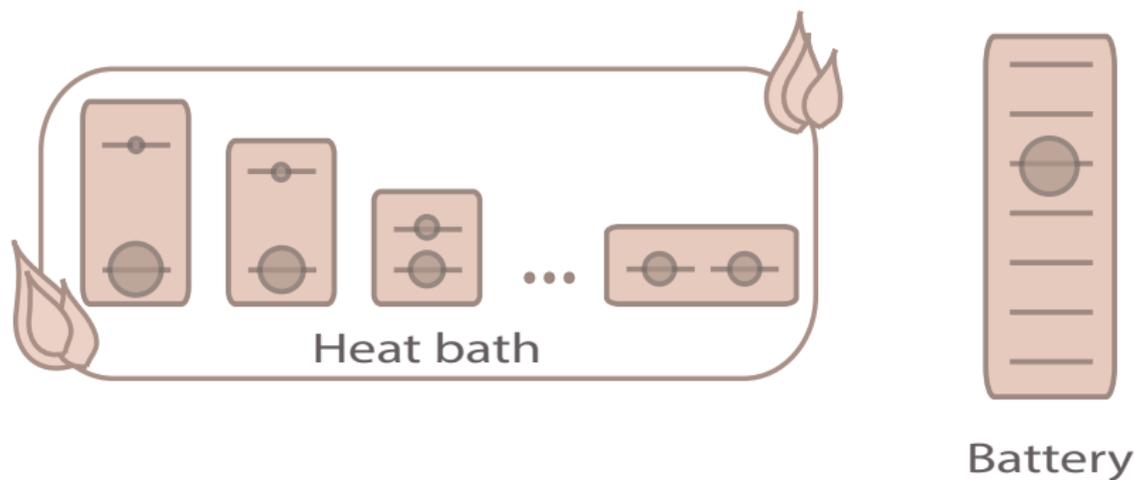
Quantum Szilard box



Semi-classical model [Alicki *et al.*]

- ▶ Manipulating H : moving energy level by δE costs δE if state is occupied
- ▶ Thermalizing: system relaxes to $G(T) = \frac{1}{2} e^{\frac{H}{kT}}$

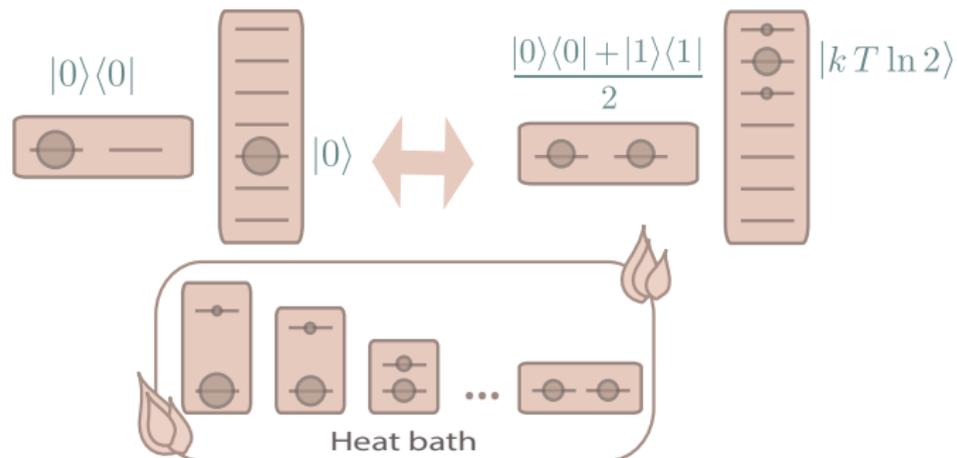
Quantum Szilard box



Quantum model [Skrzypczyk *et al.*]

- ▶ Free unitaries if $[U, H] = 0$
- ▶ Explicit heat bath and battery

Quantum Szilard box



Quantum model [Skrzypczyk *et al.*]

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Erasure with quantum side information

Memory preservation

Erase the first qubit, **preserving** the others:

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

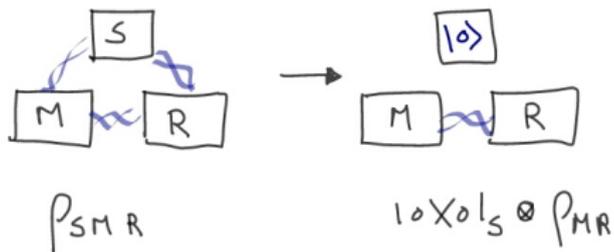
The diagram shows three qubits labeled 1, 2, and 3. A blue wavy line connects qubit 1 to qubits 2 and 3. A red bracket labeled 'S' is under qubit 1, and a red bracket labeled 'M' is under qubits 2 and 3.

$$|\psi\rangle\langle\psi|_{S1} \otimes \rho_{2,3} \rightarrow |0\rangle\langle 0|_S \otimes \underbrace{\frac{\mathbb{1}_1}{2} \otimes \rho_{2,3}}_{\rho_M}$$

Erasure with quantum side information

Memory preservation

Generally: Erase S , preserving M (and correlations)



Erasure with quantum side information

We can still use the memory optimally:

$$W(S|M) = H(S|M) kT \ln 2$$

where $H(S|M) = H(SM) - H(M)$.¹

¹[LdR et al. 2011]

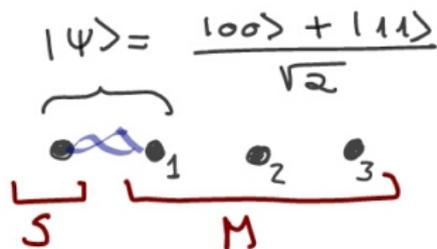
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Example



$$\underbrace{|\psi\rangle\langle\psi|_{S1} \otimes \rho_{2,3}}_{H(S|M)=-1} \rightarrow |0\rangle\langle 0|_S \otimes \underbrace{\frac{\mathbb{1}_1}{2} \otimes \rho_{2,3}}_{\rho_M}$$

¹[LdR et al. 2011]

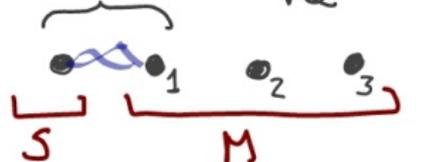
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Example

$$|\Psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$


$$|\Psi\rangle\langle\Psi|_{S1} \rightarrow \frac{1_S}{2} \otimes \frac{1_1}{2} + \text{work } 2kT \ln 2$$

¹[LdR et al. 2011]

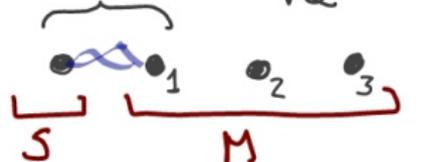
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Example

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$


$$\frac{1}{2} \rightarrow |0\rangle\langle 0|_S - \text{work } kT \ln 2$$

¹[LdR et al. 2011]

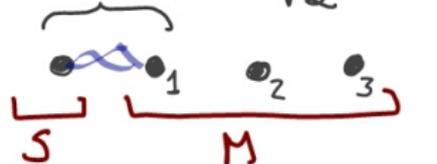
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Example

$$|\psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$


Total: $W(S|M) = -kT \ln 2 = H(S|M) kT \ln 2$

¹[LdR et al. 2011]

Work cost of computations

Cost of implementing a map \mathcal{E}

- ▶ $\mathcal{E} : X \rightarrow X'$
- ▶ unitary dilation $X \rightarrow X' \otimes E$
- ▶ $W = H(E|X')_{\mathcal{E}(\rho)} kT \ln 2$ ²

²[Faist et al. 2015]

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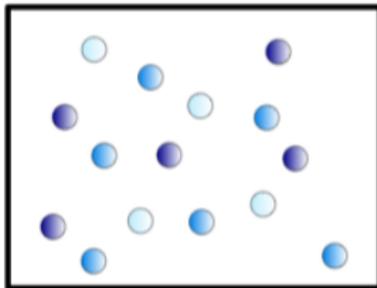
In numbers

- ▶ AND gate: $1.6 kT \ln 2$
- ▶ Running a 20 Petaflops computation: $1W$

²[Faist et al. 2015]

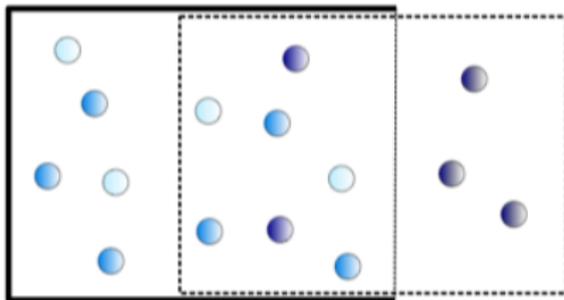
von Neumann entropy [1932]

Goal: erasure $(\sum_k p_k |\phi_k\rangle\langle\phi_k|)^{\otimes N} \rightarrow |\phi_1\rangle^{\otimes N}$



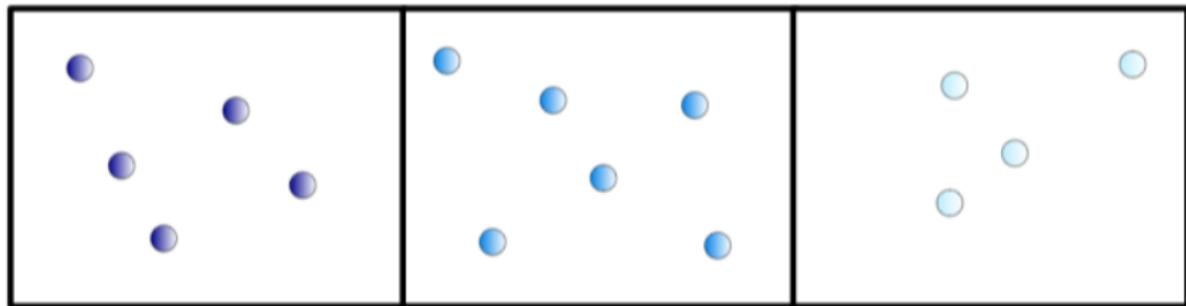
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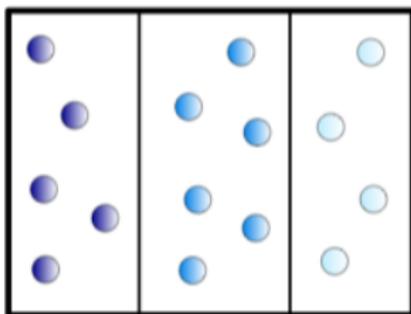
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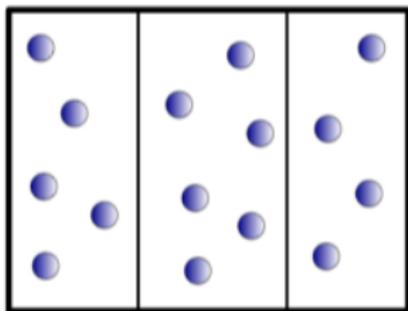
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$$V_k = p_k V \implies W_k = N_k \ln(V_k/V) = N p_k \ln p_k$$
$$\frac{W}{N} = \sum_k p_k \ln p_k \implies S(\rho) = -\text{Tr}(\rho \ln \rho)$$

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Why is thermodynamics so effective?

- ▶ ~~microscopic details~~
- ▶ Identifies:
 - ▶ easy and hard operations
 - ▶ freely available resources

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- ▶ Efficient exploitation:
 - ▶ steam engines, fridges
 - ▶ cost of state transformations

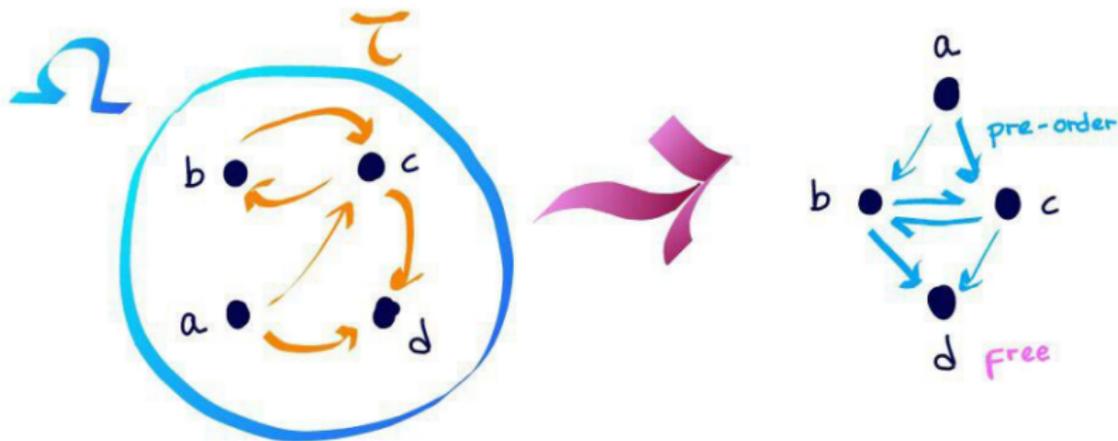
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- ▶ Operational approach: **resource theory**

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- ▶ Operational approach: **resource theory** (just like LOCC)

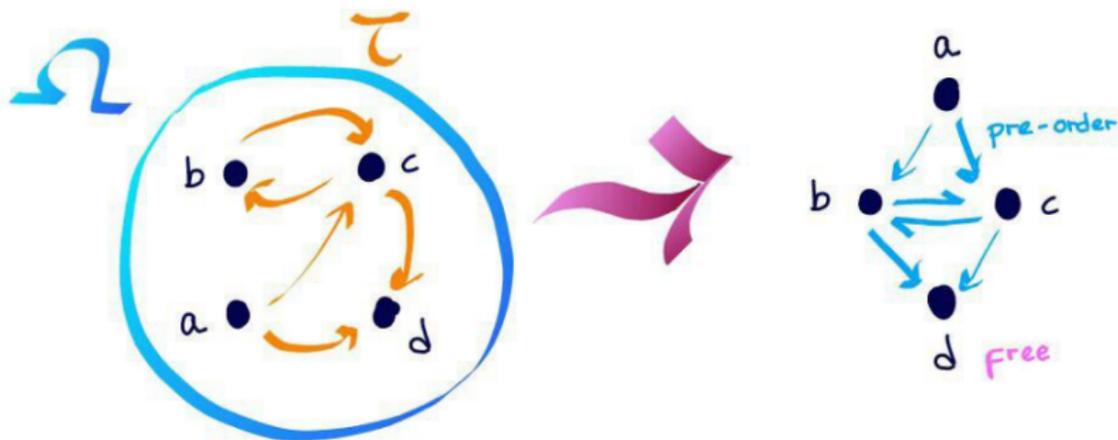
Resource theories



Operational questions

- ▶ Can one achieve $X \rightarrow Y$?
- ▶ Monotones: characterizing pre-order
- ▶ Useful and useless resources?

Resource theories



Example: LOCC

- ▶ **Allowed operations:** local operations and classical communication
- ▶ **Monotones:** formation and distillation entanglement, squashed entanglement. . .
- ▶ **Free resources:** separable states. **Currency:** Bell states

Thermodynamics as a resource theory

- ▶ Limitations:

- ▶ lack of knowledge: (N, V, T) , (N, V, E) , ...
- ▶ conservation laws: energy, momentum, ...
- ▶ limited control of operations

³[Carathéodory 1909] [Giles 1964] [Lieb and Yngvason 1998, 1999, 2003]

Thermodynamics as a resource theory

- ▶ **Limitations:**
 - ▶ lack of knowledge: (N, V, T) , (N, V, E) , ...
 - ▶ conservation laws: energy, momentum, ...
 - ▶ limited control of operations
- ▶ **Resources:** macroscopic descriptions of systems (hot gas, cold bodies)
- ▶ **Operations:** adiabatic, isothermal, ...
- ▶ **Insights:** laws of thermodynamics, free energy as a monotone, Carnot efficiency, ...³

³[Carathéodory 1909] [Giles 1964] [Lieb and Yngvason 1998, 1999, 2003]

Thermal operations

- ▶ Resources: quantum descriptions of systems (ρ_S, H_S)

⁴[Janzing 200] [Brandao *et al* 2011]

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Thermal operations

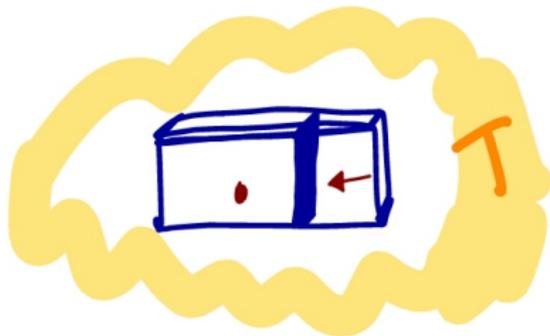
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- ▶ Toy model⁴

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Why thermal states?

$$G(T) = \frac{1}{Z} e^{-\frac{H}{kT}}$$

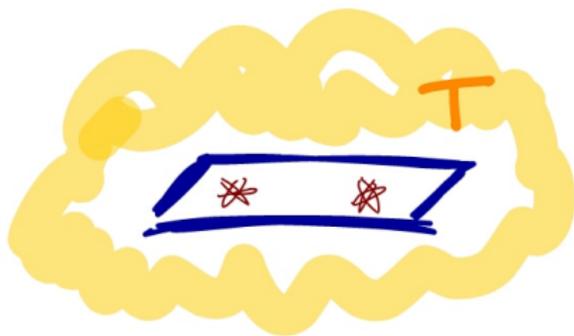
Landauer's principle



Why thermal states?

$$G(T) = \frac{1}{Z} e^{-\frac{H}{kT}}$$

Noise models



Why thermal states?

$$G(T) = \frac{1}{Z} e^{-\frac{H}{kT}}$$

Reduced description

- ▶ Large system composed of independent parts: $H = H_S + H_E$
- ▶ Energy shell Ω_E of fixed energy
- ▶ Global state of maximal entropy: $\mathbb{1}_{\Omega_E}/d_{\Omega_E}$
- ▶ $\rho_S = G_S(T(E))$

Why thermal states?

$$G(T) = \frac{1}{Z} e^{-\frac{H}{kT}}$$

Typicality of thermalization⁵

- ▶ $d_S \ll d_\Omega$
- ▶ **Static thermalization:** for most global states and most subsystems,

$$\rho_S \approx \text{Tr}_E(\mathbb{1}_\Omega/d_\Omega) = G_S(T)$$

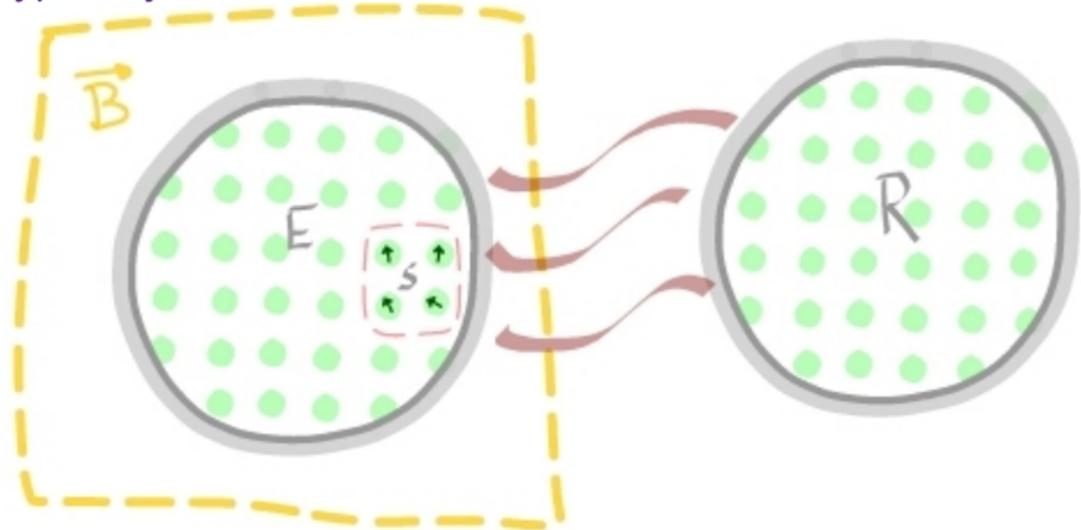
- ▶ **Decoupling** (also with side quantum information).
- ▶ Also if S corresponds to observable.

⁵Review: [Gogolin & Eisert 2016]

Why thermal states?

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Typicality of thermalization⁵



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Why thermal states?

$$G(T) = \frac{1}{Z} e^{-\frac{H}{kT}}$$

Typicality of thermalization⁵

- ▶ Evolution towards thermal state: if H is rich, $\rho_S(t) \approx G_S(T)$ for most times t and initial states.
- ▶ Time scales: under study

⁵Review: [Gogolin & Eisert 2016]

Why thermal states?

$$G(T) = \frac{1}{Z} e^{-\frac{H}{kT}}$$

Complete passivity

Intuition: only free state that does not trivialize the resource theory

- ▶ Allowed operations: unitaries
- ▶ Allowed many copies of a state
- ▶ Cannot extract energy $\implies G(T)^{\otimes n}$

Why thermal states?

$$G(T) = \frac{1}{Z} e^{-\frac{H}{kT}}$$

Complete passivity

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- ▶ Cannot extract energy $\implies G(T)^{\otimes n}$

Still a spherical cow...

Insights: noisy operations

Case $H = 0$

- Pre-order: majorization⁶ $\rho \rightarrow \sigma \iff \rho \prec \sigma$,

$$\rho \prec \sigma \iff \sum_{i=1}^k \lambda_i(\rho) \geq \sum_{i=1}^k \lambda_i(\sigma), \forall k$$

⁶Review: [Gour et al (2013)]

Insights: noisy operations

Case $H = 0$

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- ▶ Monotones: **Schur-convex functions**, e.g.

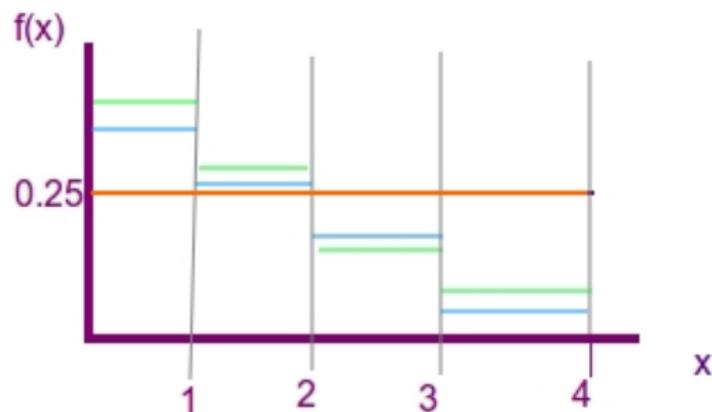
$$D^\alpha(\rho_S || \mathbb{1}_S/d_S),$$

entropies $H(\rho), H_\alpha(\rho), \dots$

- ▶ Classically: $D^\alpha(\rho || \sigma) = \frac{\text{sgn } \alpha}{\alpha-1} \log \sum_i p_i^\alpha q_i^{1-\alpha}$

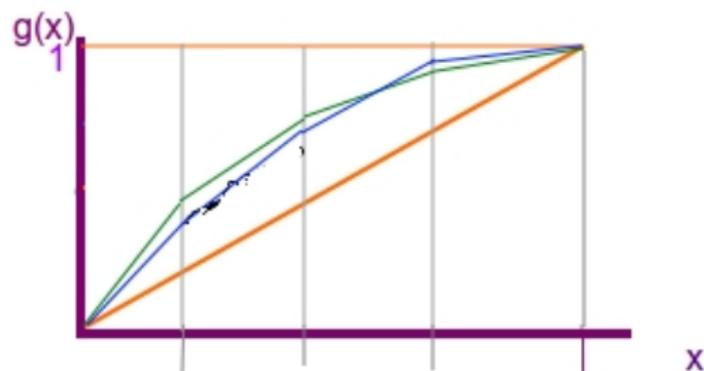
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Lorenz curves



1. Sort eigenvalues of ρ : $p_1 \geq p_2 \geq \dots \geq p_d$
2. Build step function: $f_\rho(x) = p_i(\rho)$ for $i - 1 \leq x < i$

Lorenz curves



1. Integrate to get Lorenz curve:

$$g_{\rho}(x) = \int_0^x f_{\rho}(x') dx'$$

2. Pre-order: $\rho \rightarrow \sigma \iff g_{\rho}(x) \geq g_{\sigma}(x), \forall x \in [0, 1[$

Insights: thermal operations

General Hamiltonian

- ▶ Pre-order: thermo-majorization (for block-diagonal states!)
- ▶ Monotones: e.g. relative entropy to thermal state

$$D^\alpha(\rho||G(T)),$$

free energies. . .

- ▶ Rescaled Lorenz curves⁷

⁷[Renes] [Horodecki & Oppenheim]

Insights: thermal operations

Rescaled Lorenz curves

For block-diagonal states,

1. Rescale eigenvalues: $r_i = p_i e^{\beta E_i}$ and sort them.
2. Build step function:

$$f_\rho(x) = r_i \quad \text{for} \quad \sum_{k < i} e^{-\beta E_k} \leq x < \sum_{k \leq i} e^{-\beta E_k}$$

3. Integrate to get Lorenz curve:

$$g_\rho(x) = \int_0^x f_\rho(x') dx'$$

4. Pre-order: $\rho \rightarrow \sigma \iff g_\rho(x) \geq g_\sigma(x), \forall x \in [0, 1)$

Recovering the second law

Free energies as monotones

► $\rho \rightarrow \sigma \implies F^\alpha(\rho, T) \geq F^\alpha(\sigma, T), \forall \alpha$, where⁸

$$F^\alpha(\rho, T) = kT [D^\alpha(\rho || G(T)) - \log Z]$$

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- ▶ Free energies: rescaling of $D^\alpha(\rho||G(T))$ such that

$$F^1(|E\rangle\langle E|, T) = E$$

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Recovering thermodynamics

Third law

Cannot cool to ground state with finite resources.⁹

⁹[Masanes & Openheim 2014] [Janzing] [Wilming (*in prep.*)]

¹⁰[LdR *et al* 2011]

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Recovering thermodynamics

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Cannot cool to ground state with finite resources.⁹

Landauer's principle

- ▶ Work cost of erasing S in the presence of M costs ¹⁰

$$W \approx kT H(S|M)_\rho$$

- ▶ Single-shot: H^ε , finite-size effects¹¹ limit efficiency

⁹[Masanes & Openheim 2014] [Janzing] [Wilming (*in prep.*)]

¹⁰[LdR *et al* 2011]

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Recovering thermodynamics

Fluctuation theorems

- ▶ Crooks' and Jarzinsky's relations: prob. violation exponentially suppressed
- ▶ Beyond two-measurement setting for coherent processes,¹²
e.g. $|+\rangle \rightarrow |0\rangle$

¹²[Elouard et al 2015] [Åberg 2016] [Perarnau-Llobet, et al (2016)]

Multiple conserved quantities

- ▶ Multiple¹³ conserved quantities A_1, A_2, \dots
- ▶ Allowed operations: $U : [U, A_i] = 0, \forall i$

¹³ [Vaccaro and Barnett 2011] [Lostaglio et al 2015] [Guryanova et al 2015]
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- ▶ Typical thermalization and passivity results hold
- ▶ Monotones: $D^\alpha(\rho || G(\beta_1, \beta_2, \dots))$
- ▶ First protocols for conversion between A_1, A_2, \dots

¹³ [Vaccaro and Barnett 2011] [Lostaglio et al 2015] [Guryanova et al 2015] [Yunger Halpern et al 2015] [Perarnau-Llobet et al 2015]

Coherence: quantum-quantum thermodynamics?

Needed to implement unitaries (laser).

¹⁴[Åberg 2014]

¹⁵[Korzekwa et al 2016]

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- ▶ Relevant properties of coherence reservoir ρ : (Δ_ρ, M_ρ)
 - ▶ $\Delta_\rho = \text{Tr}(\frac{1}{2}(\Delta + \Delta^\dagger)\rho)$ coherence: $\Delta = \sum_n |n+1\rangle\langle n|$
 - ▶ M : lowest occupied energy level

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 - ▶ no degradation of Δ
 - ▶ can always pump up M with energy
- ▶ More realistic reservoirs:¹⁵
 - ▶ protocol for work extraction from coherent states
 - ▶ operationally restoring reservoir: $(\Delta_\rho, M_\rho) = (\Delta_{\rho'}, M_{\rho'})$

¹⁴[Åberg 2014]

¹⁵[Korzekwa et al 2016]

Clocks and control

$$[U, H_0] = 0, \quad U = e^{-i t H_U}$$

- ▶ $H_U(t) \implies$ control
- ▶ Effort of building and keeping control systems
- ▶ Clocks and controls are out of equilibrium systems
- ▶ Fairer book-keeping: give agents little control, explicit clocks

¹⁶[Brandao et al (2011)] [Malabarba et al 2014]

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First steps

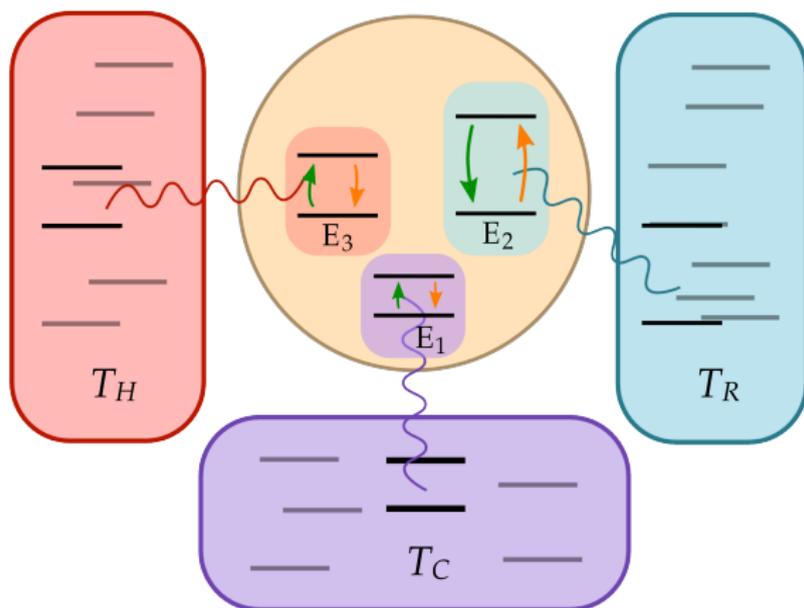
- ▶ Ideal clock (particle in a line) \implies catalytic, perfect U ¹⁶
- ▶ Thermal contact¹⁷
- ▶ Dimension bounds and clock degradation¹⁸

¹⁶[Brandao et al (2011)] [Malabarba et al 2014]

¹⁷[Wilming et al 2014]

¹⁸[Woods et al 2016]

Autonomous thermal engines



Carnot efficiency¹⁹ (fine-tuned gaps)

¹⁹[Skrzypczyk et al (various)]

Open questions

Clocks and control

- ▶ Designs for efficient clocks (theory and experiment)
- ▶ Combine with insights from reference frames
- ▶ Further restrictions (no fine-tuning of baths, Hamiltonians)
- ▶ Clean framework (how much control to give the agent?)
- ▶ Relation to coherence (again!)
- ▶ Relation to time in foundations

Open questions

Realistic resource theories

- ▶ Operational notions of temperature: baths beyond Gibbs ²⁰
- ▶ Finite-size effects ²¹
- ▶ Beyond weak coupling
- ▶ Realistic resource descriptions for experimentalists ²²
- ▶ Towards operational resource theories²³

²⁰[Farshi et al (in prep)]

²¹[Reeb et al (2013)], [Woods et al (2015)]

²²[LdR et al (2015)], [Krämer & LdR (2016)]

²³[Yunger Halpern (2015)]

Open questions

Generalized probability theories

- ▶ GPTs: apply von Neumann's operational approach to entropy²⁴
- ▶ Relate thermodynamics on different physical theories

AdS/CFT

- ▶ Notions of thermalization
- ▶ Black hole entropy & information paradox

²⁴[Barnum et al 2015]

Thank you for your attention!

Reviews

- ▶ Goold, Huber, Riera, LdR & Skrzypczyk, The role of quantum information in thermodynamics — a topical review, J. Phys. A, **49**, 14 (2016).
- ▶ Gour, Müller, Narasimhachar, Spekkens, Yunger Halpern, The resource theory of informational nonequilibrium in thermodynamics, Phys. Rev. Lett. 111, 250404 (2013).
- ▶ Gogolin, Eisert, Equilibration, thermalisation, and the emergence of statistical mechanics in closed quantum systems, Rep. Prog. Phys. 79, 056001 (2016).
- ▶ Binder, Correa, Gogolin, Anders, Adesso (editors) Thermodynamics in the quantum regime: Fundamental aspects and new directions, Springer (2018).²⁵

²⁵Individual chapters present on arXiv.